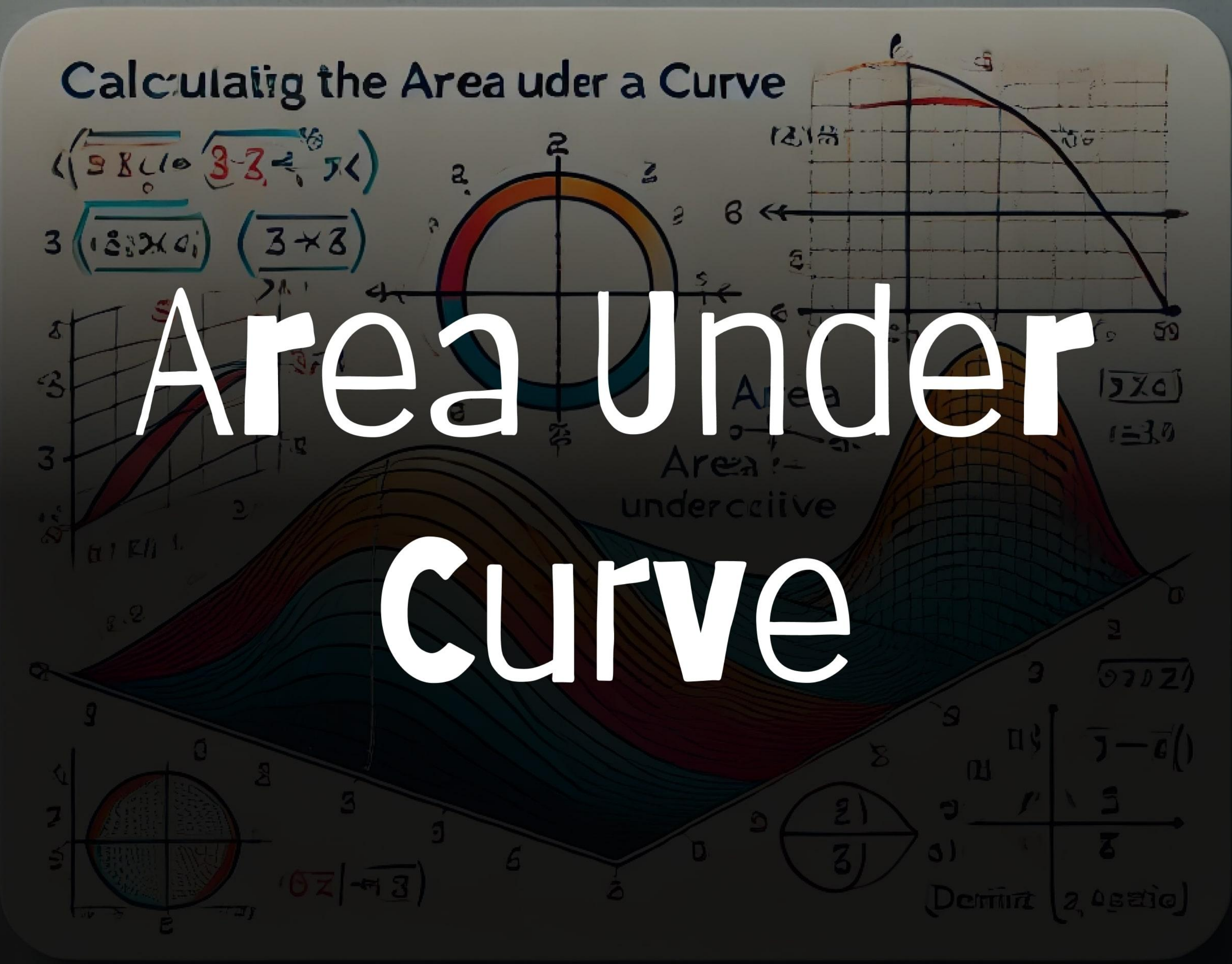
Area Under Curve



AREA UNDER CURVE

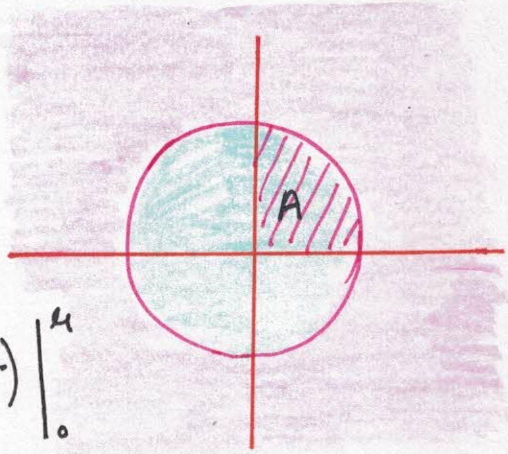
Area under curve always calculates the area of closed region. In this chapter we deal the area bounded by $f(x)$ with either some other curve eq some boundary.

Here, the sum of area will not be in algebrical, it will be geometrical i.e. always comes +ve whether the bounded region lies above or below x axis.

Here, note that all the boundaries must lie in the bounded region.

NOTE: With the help of AUC, we can also find the area of expression which are not function.

Question Find the area of $x^2+y^2=r^2$



$$y^2 = r^2 - x^2 \Rightarrow y = \pm\sqrt{r^2-x^2}$$

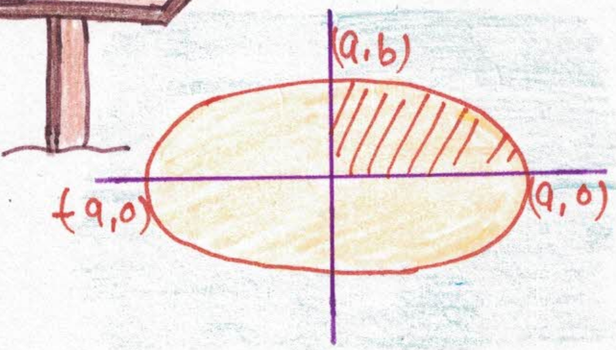
$$A = \int_{-r}^{r} \sqrt{r^2-x^2}\cdot dx$$

$$A = \frac{x}{2}\sqrt{r^2-x^2} + \frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right)\Big|_0^r$$

$$A = \frac{r^2}{2}\left(\pi/2\right)$$

$$\text{area of circle} = 4A = \boxed{4\left(\frac{r^2\pi}{4}\right) = \pi r^2}$$

Question Find area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$$y^2 = \left(1-\frac{x^2}{a^2}\right)b^2$$

$$y = \pm \frac{b}{a}\sqrt{a^2-x^2}$$

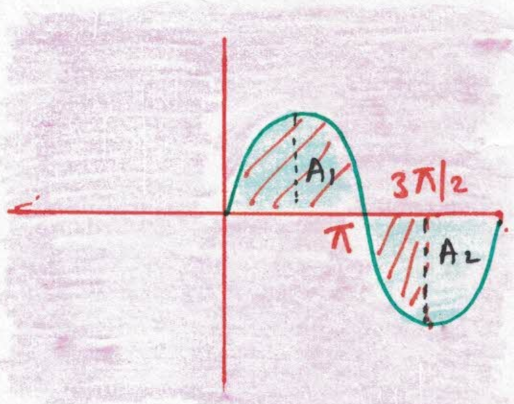
$$A = \int_{-a}^{a} \frac{b}{a}\sqrt{a^2-x^2}\, dx$$

$$A = \frac{2b}{a}\left(\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)\Big]_0^a$$

$$A = \frac{2b}{a}\left(\frac{a^2\pi}{4}\right)$$

Area of ellipse $= 2A$ $\boxed{= \pi ab}$

Question Find the area bounded by the curve $y=\sin x$ b/w $x=0$ and $x=3\pi/2$ & x-axis



$$|A_1| = \int_0^{\pi} \sin x \cdot dx$$

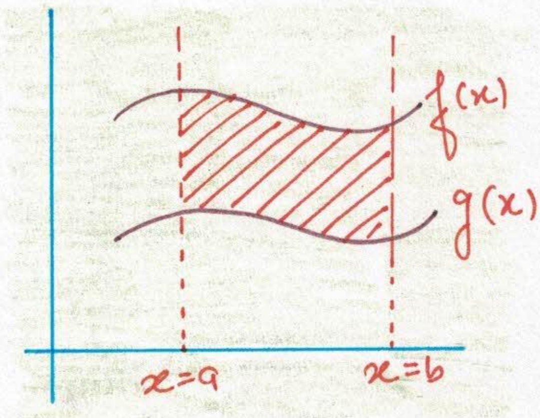
$$= -\cos x \Big|_0^{\pi} = 1+1 = 2$$

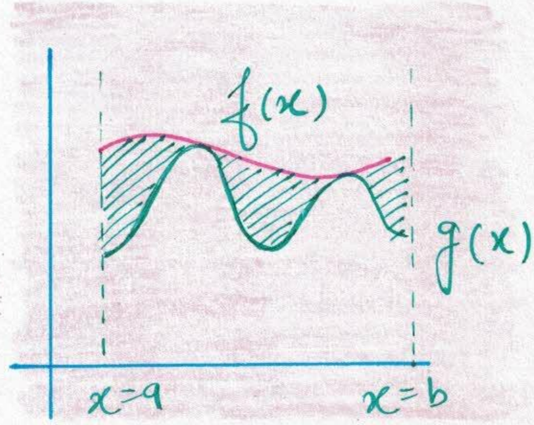
$$|A_2| = -\cos x \Big|_{\pi}^{3\pi/2} = 1$$

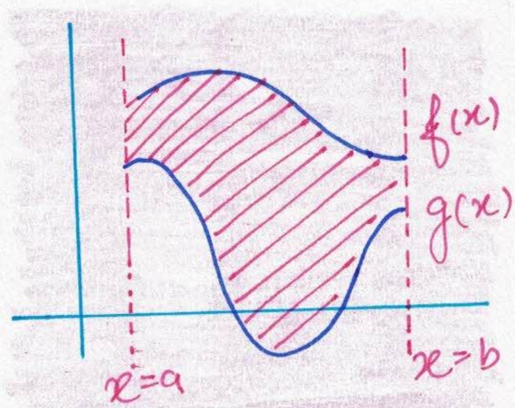
$\boxed{\text{Area} = 3 \text{ sq. units}}$

TYPE-II Area bounded by $f(x)$ and $g(x)$ can be seen in 2 ways b/w $x=a$ & $x=b$.

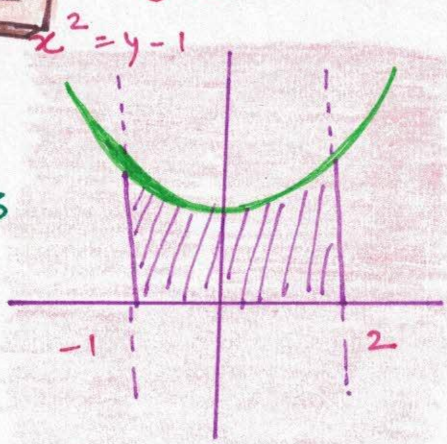
$$\int_a^b \left| f(x) - g(x) \right| \cdot dx$$







Find the area bounded by the curve $y = x^2+1$ x-axis and the lines $x=-1$ and $x=2$



$$\int_{-1}^{2} (x^2+1)\,dx$$

$$\left[\frac{x^3}{3}+x\right]_{-1}^{2}$$

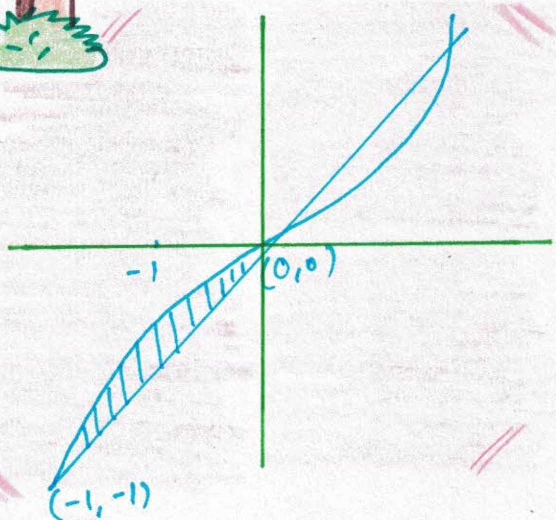
$$\boxed{=6}$$

Question

$y = x \quad , \quad y = x^3$ in 3rd quad.

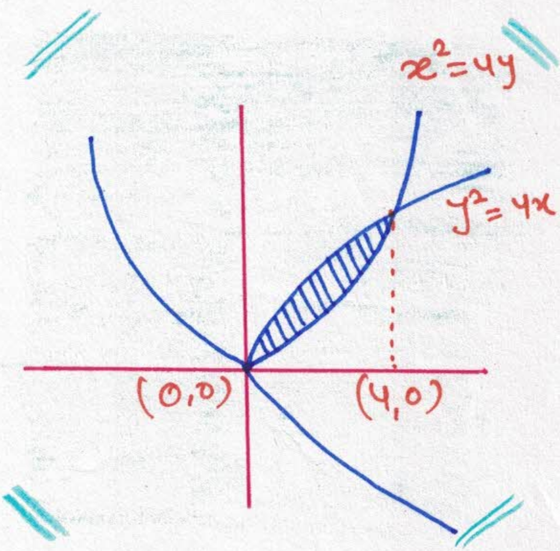
$$\int_{-1}^{0} x \cdot dx - \int_{-1}^{0} x^3 \cdot dx \Rightarrow \frac{x^2}{2} - \frac{x^4}{4}$$

$$\boxed{= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}}$$



Question

$y^2 = 4x$ and $x^2 = 4y$



$$4(\sqrt{4y}) = y^2$$

$$16(4y) = y^4$$

$$y^4 - 64y = 0$$

P.O.I (4,0) $\quad y(y^3 - 64) = 0$

$$y = 0 \qquad y = (64)^{1/3}$$

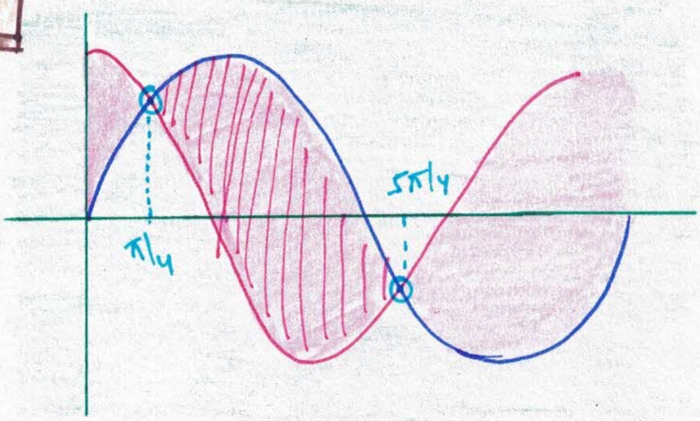
$$x = 4$$

$$\int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) \cdot dx = \frac{2x^{3/2}}{3/2} - \frac{x^3}{4 \times 3}\Bigg]_0^4$$

$$\boxed{= \frac{2(2)^3}{3/2} - \frac{4^3}{12} = \frac{16}{3}}$$

Question

$y = \sin x \qquad y = \cos x$ in $[\pi/4, 5\pi/4]$.



$$\int_{\pi/4}^{5\pi/4} \sin x - \cos x \cdot dx$$

$$= -\cos x - \sin x \Big]_{\pi/4}^{5\pi/4}$$

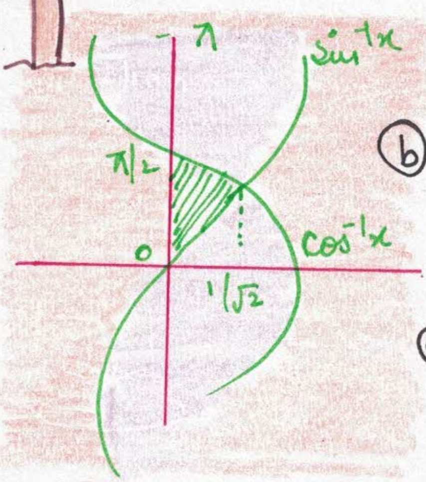
$$= -\left[\left(\cancel{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{2}}\right) - \left(\cancel{\frac{1}{\sqrt{2}}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$\boxed{= -\left[\frac{2 \times 2}{\sqrt{2}}\right] = 2\sqrt{2}}$$

Question $y = \sin^{-1}x \quad , y = \cos^{-1}x$

(a) with x axis

(b) with y axis



(b) $$Y = \int_0^{1/\sqrt{2}} \cos^{-1}x - \sin^{-1}x \cdot dx$$

(a) $$\text{Area} = \int_0^{1/\sqrt{2}} \sin^{-1}x \cdot dx + \int_{1/\sqrt{2}}^{1} \cos^{-1}x \cdot dx$$

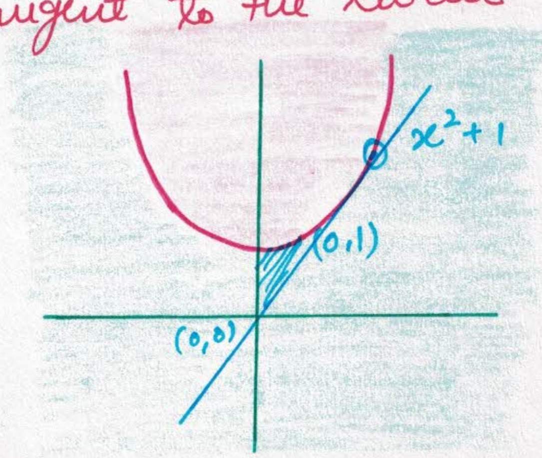
Question find the area bounded by the curve $y = x^2+1$ x, axis and y axis and tangent to the curve at $(3, 10)$.

$y + y_1 = xx_1 + 1$

$(y+10) = 2(3x+1)$

$y - 10 = 6x + 2$

$$\boxed{y = 6x + 8 \qquad P = \left(\frac{-4}{3}\right)}$$



$$A_1 = \int_0^{4/3} (x^2+1)\cdot dx \; - \int 6x-8$$

$$= x^3 + x \Big]_0^{4/3} - \Big|^{4/3} \frac{\cancel{6}^{3}x^2}{\cancel{2}} - 8x$$

$$= \frac{64}{3} + \frac{4}{3} = \frac{68}{3} - \left(\frac{3\times 16}{3}\right)$$

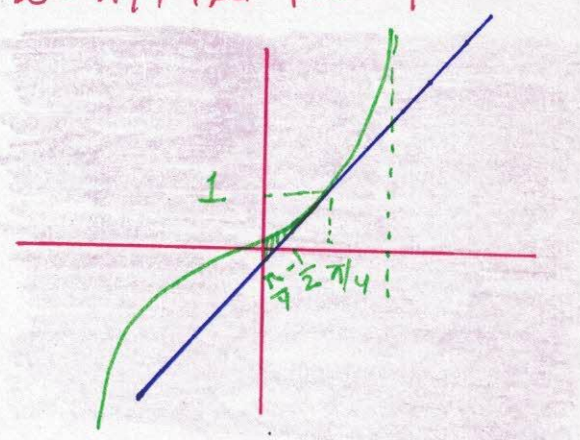
$$= \frac{68}{3} - \frac{48-32}{3} = \frac{68-16}{3} = \frac{52}{3} = 9$$

$$\text{Area of } \Delta = \frac{1}{2} \times \frac{4}{3} \times 4 = \frac{16}{3}$$

$$\boxed{\text{area} = 9 - \frac{16}{3} = \frac{11}{3}}$$

Question

Find area bounded by $y = \tan x$, x axis and tangent to the curve at $x = \pi/4$ in first quadrant.



$$(y-1) = 2(x - \pi/4)$$

$$y = 2x - \pi/2 + 1$$

$$x = \frac{\pi/2 - 1}{2}$$

$$\int_0^{\frac{\frac{\pi}{2}-1}{2}} \tan x - (2x - \pi/2 + 1)\cdot dx$$

$$= \Big| \ln|\sec x| - x^2 + \frac{\pi x}{2} - x \Big|_0^{\frac{\pi/2-1}{2}}$$

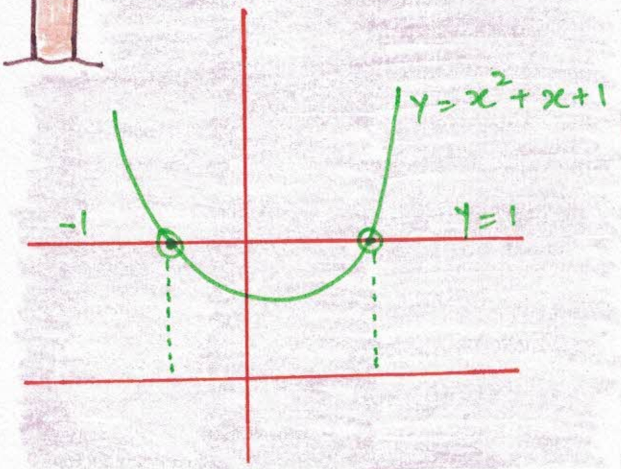
$$\text{Area under } \tan x = \int_0^{\pi/4} \tan x \cdot dx$$

$$= \ln|\sec x| \Big]_0^{\pi/4} = \ln\sqrt{2}$$

Area of Small $\Delta = \frac{1}{2} \times \frac{1}{2} \times 1.$

Req. area under curve $\boxed{= \ln\sqrt{2} - \frac{1}{4}}$

Question Find the area bounded by the curve $y = x^2 + x + 1$ and the line $y = 1$.



$$y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = y - \frac{3}{4} \qquad \left(-\frac{1}{2}, \frac{3}{4}\right)$$

$$x^2 + x = 0$$

$$x(x+1) = 0 \qquad (-1, 0)$$

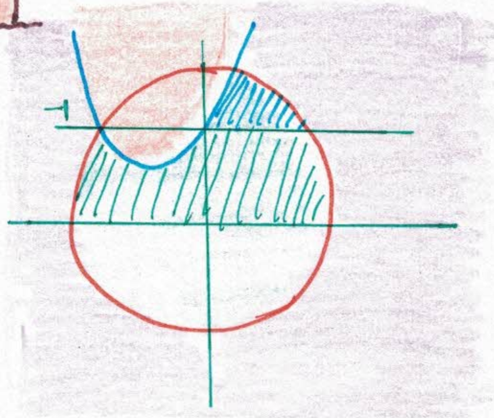
$$2 \times 1 = 2$$

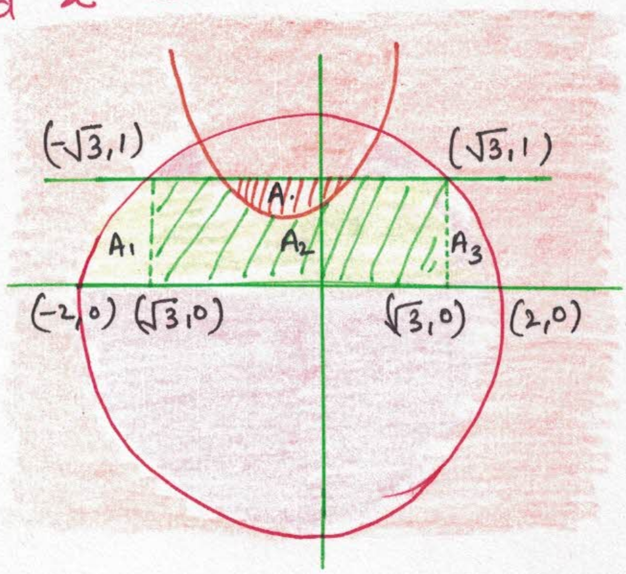
$$\int_{-1}^{0} x^2 + x + 1 \cdot dx = \frac{x^3}{3} + \frac{x^2}{2} + x \Big]_{-1}^{0}$$

$$= \frac{1}{3} + \frac{1}{2} - 1 \quad \left(-\frac{1}{3} + \frac{1}{2}\right) \Rightarrow \frac{1}{3} + 2$$

$$\boxed{\cancel{2} - \left(\frac{1}{\cancel{3}} + \cancel{2}\right) = \frac{1}{6}}$$

Question Find the area bounded by the curve $y = x^2 + x + 1$ $y = 1$, $x^2 + y^2 = 4$ and x- axis.





$x^2 + y^2 = 4$

$y = 1$

$x^2 = 3 \quad x = \pm\sqrt{3}$

$y = \int \sqrt{4 - x^2} \cdot dx$

$= \frac{x}{2}\sqrt{4-x^3} + \frac{4}{2} \sin^{-1}\frac{x}{2} \Big]_{-2}^{-\sqrt{3}}$

$A_1 + A_3 = 2\left[-\frac{\sqrt{3}}{2} - 2\left(\pi/3\right) - \left\{2\sin^{-1}(-1)\right\}\right]$

$= 2\left[-\frac{\sqrt{3}}{2} - \frac{2\pi}{3} + \pi\right] = 2\left[-\frac{\pi}{2} + \frac{\pi}{3}\right]$

$\frac{x}{2}\sqrt{4-x^2} + 2\sin\frac{x}{2}\Big]_{-\sqrt{3}}^{\sqrt{3}}$

$\frac{1}{2} \times (2\sqrt{3})(1)$

$\sqrt{3}$

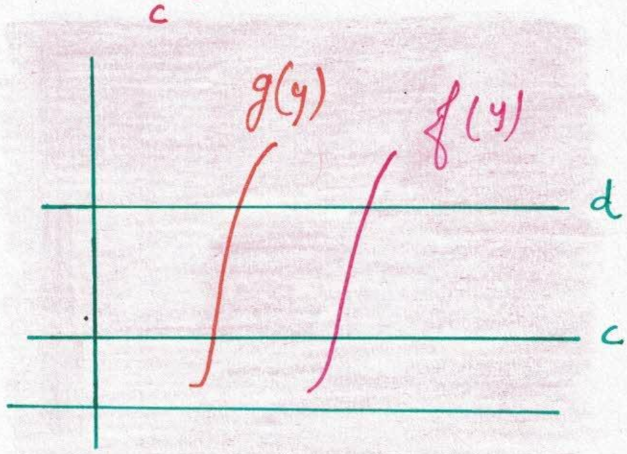
$= \sqrt{3} + \frac{4\pi}{3}$

$A_1 + A_2 + A_3 = 2\sqrt{3} - \sqrt{3} + 2\pi/3$

$$\boxed{\sqrt{3} + 2\pi/3 - \frac{1}{6}}$$

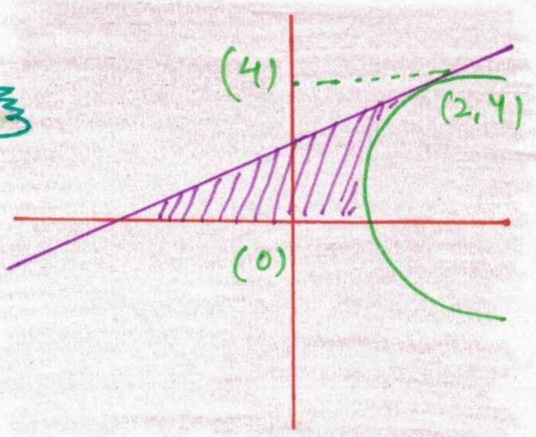
Question

If $x = f(y)$ and $x = g(y)$ are two curves, then area bounded by the curves $y = c$ and $y = d$ is given by $\int_c^d f(y) - g(y) \cdot dy$



Question

Find the area bounded by the curves $(y-2)^2 = 4(x-1)$, x axis and tangent to the curve at $(2,4)$.



$(y-2)^2 = 4(x-1)$

$\frac{(y-2)^2}{4} + 1 = x$

Tangent at $(2,4)$

$(y-2)(4-2) = 2((x-1)+(2-1))$

$y-2 = x$

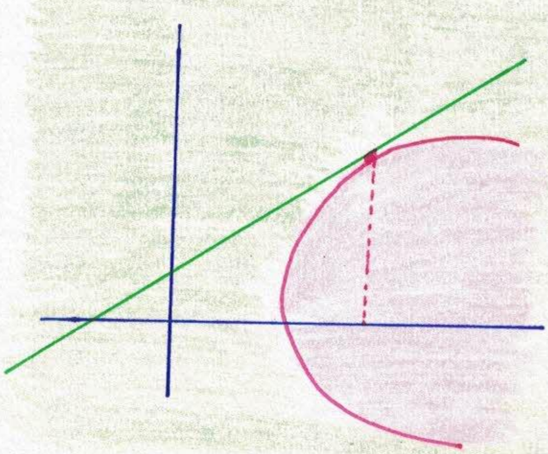
?

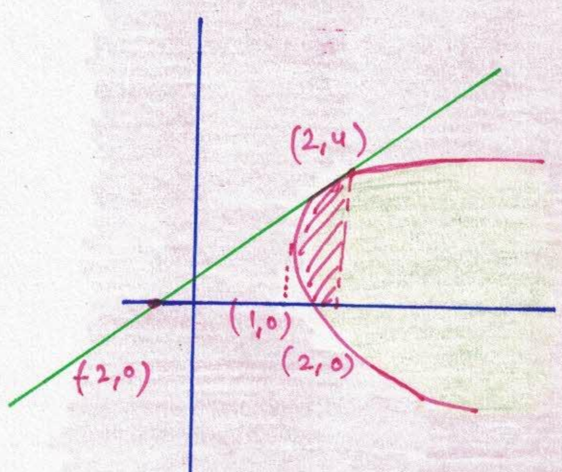
$$\int_0^4 \frac{(y-2)^2}{4} + 1 - (y-2) \cdot dy$$

$$= \frac{(y-2)^3}{3\times4} + y - \frac{y^2}{2} + 2y \Big|_0^4$$

$$= \frac{\cancel{8}^{2}}{\cancel{4}\times3} + 4 - \frac{\cancel{16}^{8}}{\cancel{2}} + \cancel{8}$$

$$\boxed{= \frac{2+12}{3} = \frac{14}{3}}$$





area of $\Delta = \frac{1}{2} \times 4 \times 4 = 8$

$(y-2)^2 = 4(x-1)$ $\quad y = 2+2\sqrt{x-1}$

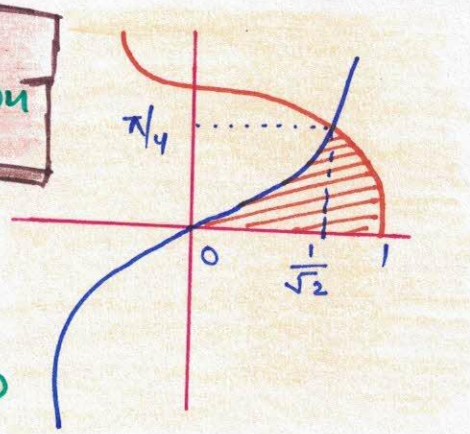
$y = 2-2\sqrt{x-1}$

$$\int_1^2 (2+2\sqrt{x-1} - (2-2\sqrt{x-1})\cdot dx$$

$$4\int_1^2 \sqrt{x-1}\ .dx$$

$$= 4 \times \frac{2}{3}(x-1)^{3/2} - 8/3$$

Req. area = $\boxed{8 - \frac{8}{3} = \frac{16}{3}}$

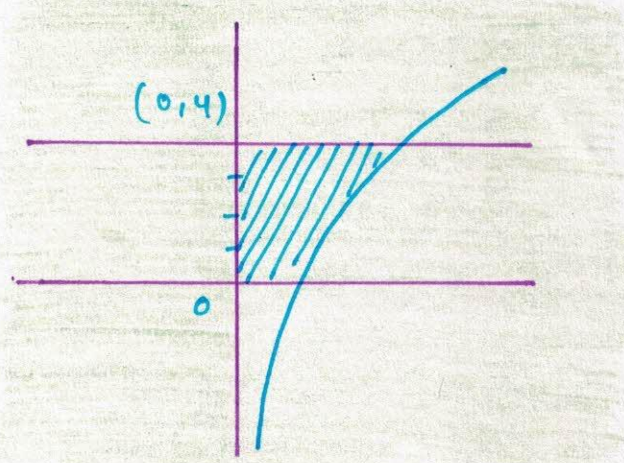


$$\int_0^{\pi/4} \cos y - \sin y \cdot dy$$

$$= \sin y + \cos y \Big|_0^{\pi/4}$$

$$\boxed{= \sqrt{2} - 1}$$

Find the area bounded by $y = \ln x$, x axis and $y = 4$.



$\ln x = 4$

$y = \ln x$

$x = e^y$

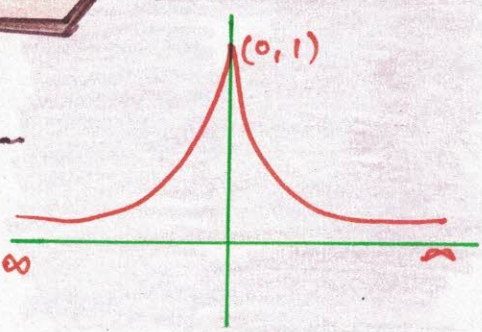
$$\int \ln e^y dy \Rightarrow e^y \Big]_0^4$$

$$\boxed{e^4 - 1}$$

TYPE - II

Sometimes, the area of consideration will not be actually closed but due to asymptote, it is supposed to be closed, then we can find the area of closed region.

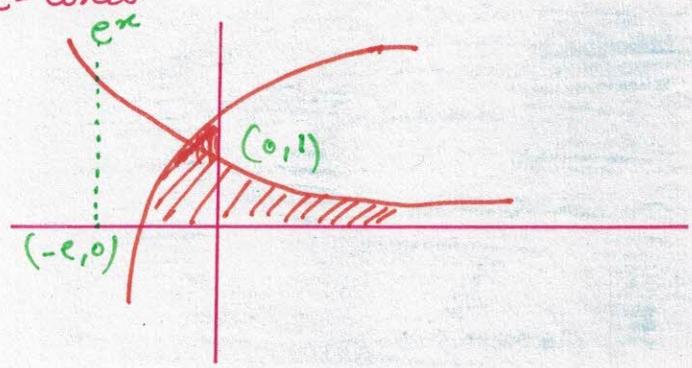
Question Find the area bounded by $f(x) = \dfrac{1}{1+x^2}$ by x axis.



$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \tan^{-1} x \Big|_{-\infty}^{\infty}$$

$$\boxed{= \frac{\pi}{2} + \frac{\pi}{2} = \pi}$$

Question Find the area bounded by the curve $y = e^{-x}$, $y = \ln(x+e)$ and x-axis.



$$\int_0^1 (-\ln y\, dy) - \int_0^1 e^y - e$$

$$= 0 - \left[e^1 - e + e\right]$$

$$= e$$

$\ln y = -x$

$x + e = e^y$

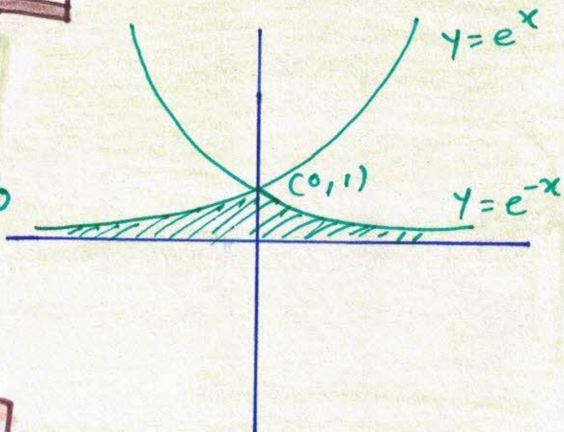
$$\int_{1-e}^{0} \ln(x+e)\,dx + \int_0^{\infty} e^{-x}\cdot dx$$

$$(x+e)\ln\left(\frac{x+e}{e}\right)\Big|_{1-e}^{0} + \left(-e^{-x}\right)\Big|_0^{\infty}$$

$$\boxed{= e + 1 + 1 = e + 2}$$

Question

Find the area bounded by $y = e^x$, $y = e^{-x}$ and x-axis.



$$\int_{-\infty}^{0} e^x \cdot dx = e^x \Big|_{-\infty}^{0} = 1$$

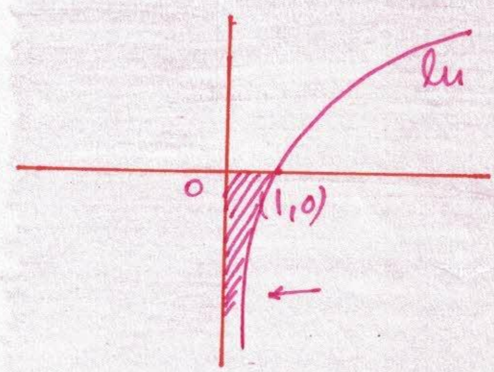
$$\boxed{2(1) = 2}$$

Question

$y = \ln x$, y-axis and x = axis

$$\int_0^1 \ln x \cdot dx = x^1$$

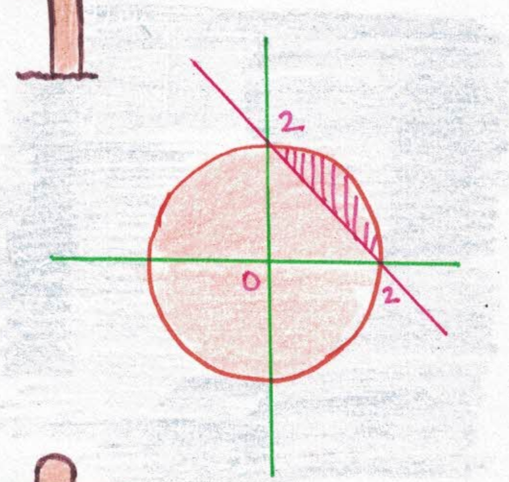
$$\int_{-\infty}^{0} e^y \cdot dy = e^y \Big|_{-\infty}^{0} = 1$$



TYPE - III

When the eq^n of the curve isn't given directly i.e. inspite of equation, inequation is given.

Question $x^2+y^2 \le 4$ $\qquad x+y>2$



$$\int_0^2 \sqrt{4-x^2} - (2-x)\cdot dx$$

$$= \int_0^2 \frac{x}{2}\left(\sqrt{4-x^2}\right) \cdot \frac{\cancel{4}^{2}}{\cancel{2}} \sin^{-1}\frac{x}{2} - \left(2x-\frac{x^2}{2}\right)$$

$$= 2(\pi/2) - (4-2) = \boxed{\pi-2}$$

Question Find the area bounded by inequalities.

$$|x|+|y| \le 4$$

$$y^2+8x \le 0$$

$y^2 = -8x$
$y^2 = -8(y-4)$
$y^2+8y+32=0$

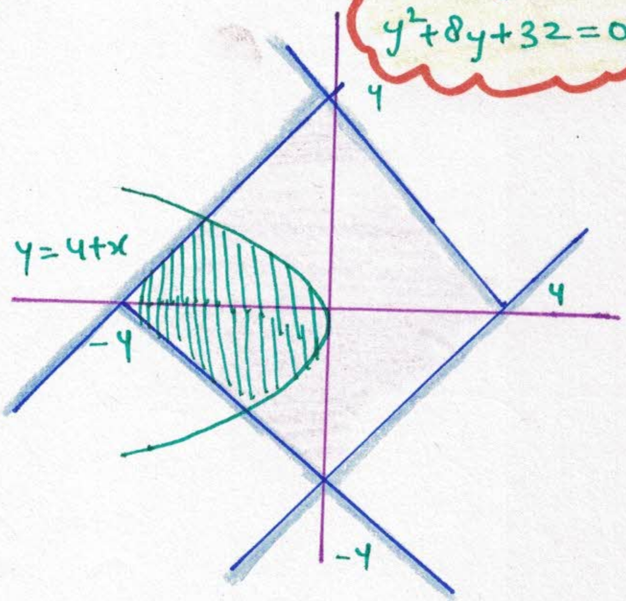
$$\int_{-4}^{0} -\frac{y^2}{8} - (y-4)\,dy$$

$$= \left. \frac{-y^3}{24} - \frac{y^2}{2} + 4y \right]_{-4}^{0}$$

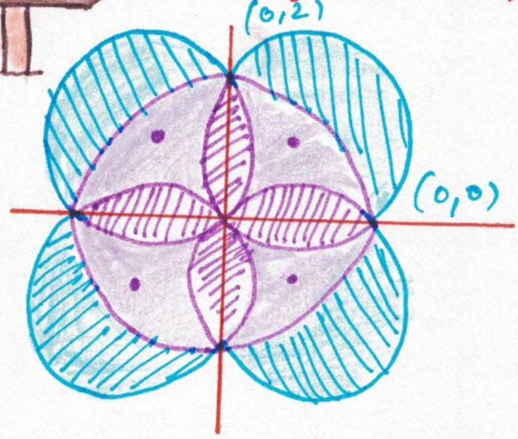
$$= \frac{64}{24} - \frac{16}{2} - 16$$

$$= \frac{\cancel{16}^{8}}{\cancel{6}_{3}} - 8 - 16 = \frac{8}{3} - 24$$

$$= \frac{8-72}{3} = \boxed{\frac{64}{3}}$$



Question $x^2+y^2 \ge 4$, $x^2+y^2-2|x|-2|y| \le 0$



$\perp . \perp \quad R = \sqrt{2}$

$4-2x-2y=0$

$2-x-y=0$

$x+y=2$

$(x-1)^2+(y-1)^2=2$

$(y-1)^2 = 2\cdot (x-1)^2$

$y = 1+\sqrt{2-(x-1)^2}$

$$\int_0^2 \left(1+\sqrt{2-(x-1)^2} - \sqrt{4-x^2}\right) . dx$$

$$x + \frac{x-1}{2}\sqrt{2-(x-1)^2} + \frac{2}{2}\sin^{-1}\frac{(x-1)}{\sqrt{2}} - \frac{x}{2}\sqrt{4-x^2} - \frac{4}{2}\sin^{-1}\frac{x}{2}\Big|_0^2$$

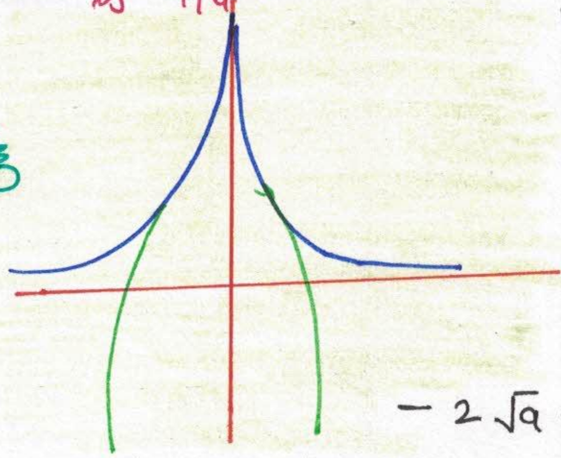
$$= 2+\frac{1}{2} + \sin^{-1}\frac{1}{\sqrt{2}} - 0 - 2\left(\pi/2\right) - \left(-\frac{1}{2} + \left(\pi/4\right)\right)$$

$$= \frac{5}{2} + \pi/4 + \pi + \frac{1}{2} - \pi/4 = 3 + \frac{3\pi}{2} - \pi$$

$$4n = 12 + \cancel{4}^{2}\left(\frac{3\pi}{\cancel{2}}\right) \qquad \boxed{12-2\pi}$$

TYPE-IV

Question For what values of 'a' the area of the fig. bounded by the curve $y = 4/x^2$ $\quad x=1$, $y=a$ is $9/4$.



$$y = \left(\frac{2}{x}\right)^2 \qquad y = 16$$

$$\int_1^{2/\sqrt{a}} \frac{4}{x}\,.dx - \int a\,.dx$$

$$\frac{-4}{x}\Big|_1^{2/\sqrt{a}} - ax\Big|_1^{2/\sqrt{a}}$$

$$-2\sqrt{a} + a\frac{\cancel{4}^{2}}{\cancel{2}}\sqrt{a} + 4 = \frac{9}{4}$$

$$\int_1^{2/\sqrt{a}} \frac{4}{x^2}\,.dx \qquad -\sqrt{2a} + a - 2\sqrt{a} = -\frac{7}{4}$$

$$a + -4\sqrt{a} = -7/4$$

$$a = \frac{1}{4}, \frac{49}{4} \qquad t^2 - 4t + \frac{7}{4} = 0$$

$$4t^2 - 16t + 7 = 0$$

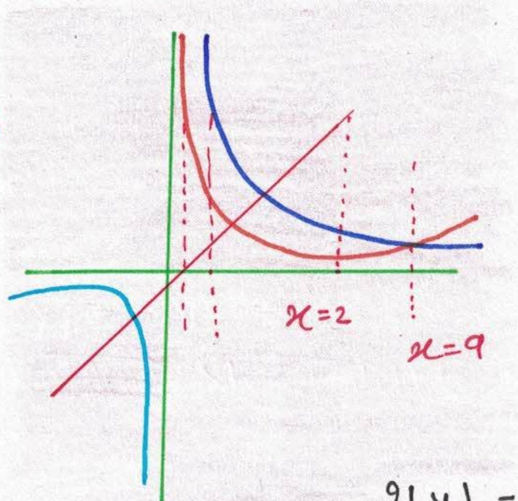
$4t^2 - 14t - 2t$

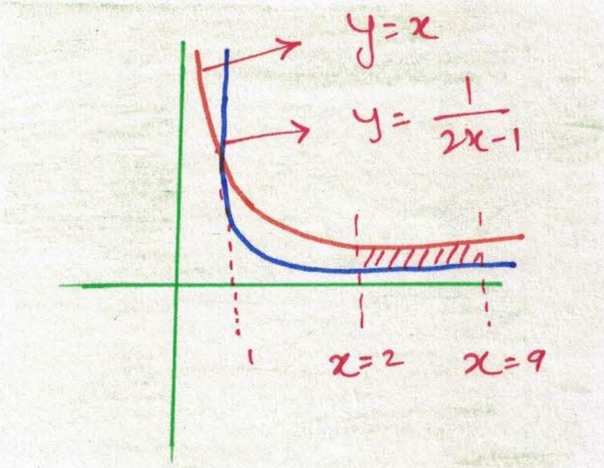
$2(2t-7) - 2(t-7)$

$$\boxed{t = 7/2 \qquad t = 7}$$

Question

For what values of a. the area of the fig. bounded by the lines, $x=2$, $x=9$ with the curve $y=\frac{1}{x}$ and $y=\frac{1}{2x-1}$ will be $\ln 4/\sqrt{5}$.





$$2(y) - (x - \tfrac{1}{2}) = 1$$

$$y = \frac{1}{2(x - 1/2)}$$

$$\int_2^9 1/x - \left(\frac{1}{2x-1}\right) \cdot dx \Rightarrow \frac{\ln}{2} - \frac{1}{2}\ln\left(x - \frac{1}{2}\right)\Big|_2^9 = \ln\frac{4}{\sqrt{5}}$$

$$= \ln\left(\frac{a}{2}\right) - \frac{1}{2}\left(\ln\left(a - \frac{1}{2}\right) - \ln\frac{3}{2}\right) = \ln\frac{4}{\sqrt{5}}$$

$$\frac{a^2}{2} - 2 = \ln\frac{4}{\sqrt{5}} + \ln\left(a - \frac{1}{2}\right)\left(\frac{3}{2}\right)$$

$$\frac{a^2}{2} - 2 = \ln\left[\left(\frac{4}{\sqrt{5}}\right)\sqrt{\left(\frac{2a-1}{2}\right)\Big/\left(3/2\right)}\right]$$

$$\frac{a^2-4}{2} = \ln\left(\frac{4}{\sqrt{5}}\sqrt{\frac{2a-1}{3}}\right)$$

$$\frac{\sqrt{3a}}{2\sqrt{2a-1}} = \frac{4}{\sqrt{5}}$$

$$\sqrt{15a} = 8\sqrt{2a-1}$$

$$15a^2 = 64(2a-1) \Rightarrow 5a^2 - 128a + 64 = 0$$

$$(a-8)(15a-8) = 0$$

$$\boxed{a = 8 \quad a = \frac{8}{15}}$$

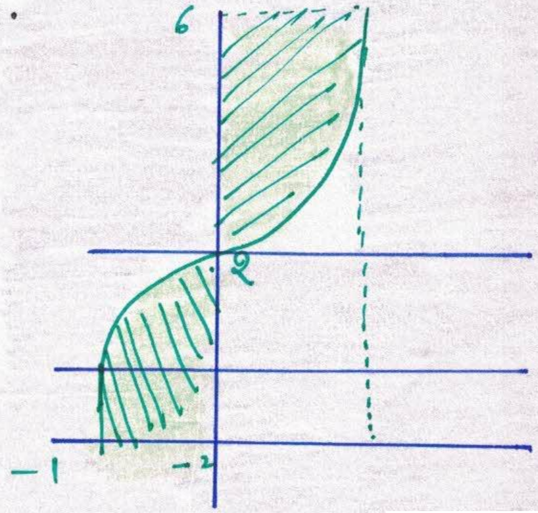
TYPE - V

Area bounded by inverse f^n of a given f^n.

Question

Find the area bounded by the inverse of a f^n $f(x) = x^3 + 3x + 2$ with x-axis, $x = 2$ and $x = 6$.

As we know a function and its inverse is always symmetrical about $y = x$ line hence, the area bounded by inverse of $f(x)$ b/w $x = -2$ and $x = 6$ with x-axis is the same area bounded by $y = f(x)$ with y-axis b/w the line $y = -2$ and $y = 6$.



$$\int_{-1}^{0} f(x) - (-2)\,dx + \int_{0}^{1} 6 - f(x)$$

$$= \left.\frac{x^4}{4} + \frac{3x^2}{2} + 4x\right|_{-1}^{0}$$

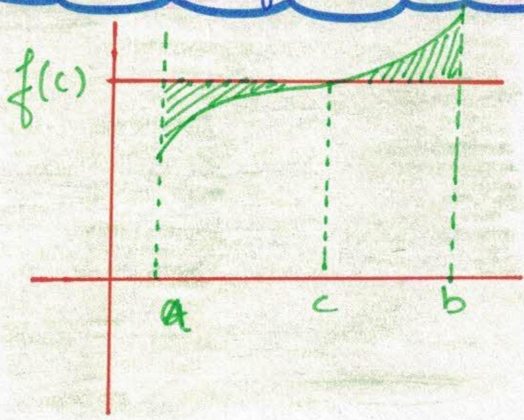
$$+ \left[6x - \frac{x^4}{4} - \frac{3x^2}{2} - 2x\right]_{0}^{1}$$

$$\left|\frac{1}{4}+\frac{3}{2}-4\right| + \left|6-\frac{1}{4}-\frac{3}{2}-2\right|$$

$$\boxed{=\frac{9}{2}}$$

VARIABLE AREA GREATEST AND LEAST VALUE

If $y=f(x)$ is a monotonic function in (a, b) then the area bounded by $x=a$, $x=b$, $y=f(x)$ and $y=f(c)$ will be min. if $c=\frac{a+b}{2}$



$$\int_a^c \left(f(c)-f(x)\right)\cdot dx+\int_c^b \left(f(x)-f(c)\right)\cdot dx$$

$$A=f(c)(c-a)-\int_a^c f(x)\,dx$$

$$+\int_c^b f(x)\,dx-f(c)(b-c)$$

$$A=f(c)(2c-a-b)+\int_a^b f(x)\cdot dx-\int_a^c f(x)\cdot dx$$

$$\frac{dA}{dc}=f'(c)\cdot(2c-a-b)+\cancel{f(c)}(2)-2\cancel{f(c)}$$

$$f'(c)(2c-a-b)=0$$

$$\Rightarrow 2c=a+b \quad \text{or} \quad \boxed{c=\frac{a+b}{2}}$$

Question

If the area bounded by $y = \frac{x^3}{3} - x^2 + a$ and the st. line $x = 0$ and $x = 2$ and the x-axis min. the find value of 'a'.

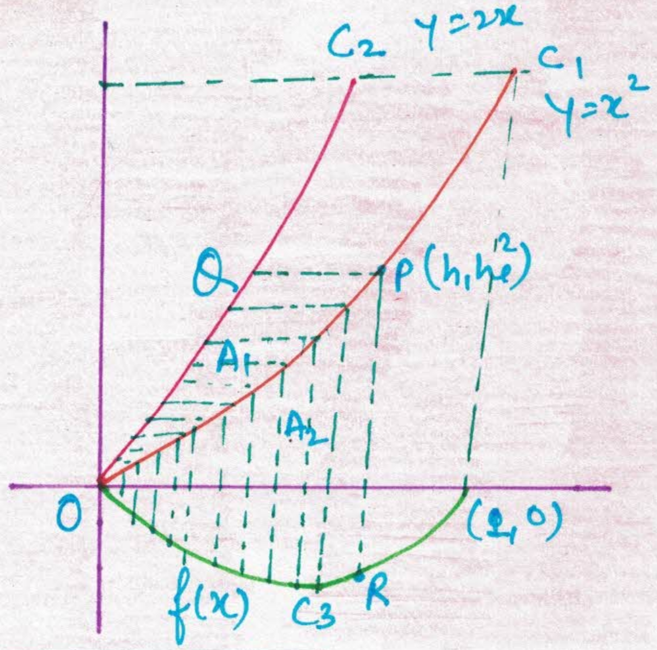
$f(1) = \frac{1}{3} - 1 + a = -\frac{2}{3} + a$ $\qquad y' = x(x-2)$

$$\int_0^1 \frac{x^3}{3} - x^2 + a \cdot dx \qquad \begin{matrix} y = 0 \\ y = f(x) \end{matrix}$$

$$= \frac{x^4}{4} - \frac{x^3}{3} + ax \Rightarrow f(1) = 0 \Rightarrow \boxed{a = 2/3}$$

DETERMINATION OF FUNCTION

Let c_1 and c_2 be the graphs of the function $y = x^2$, $y = 2x$ in $[0,1]$. Let C_3 be a graph of a f^n $y = f(x)$ in $[0,1]$ and $f(0) = 1$ for a pt. P on C_1 lines are drawn $\parallel^{el}$ to coordinate axis which meet C_2 and C_3 at Q & R respectively as shown in figure. If for every function of P on C_1 the area of the shaded region OPQ and OPR equal then determine $f(x)$.



Let $f(x) = C_3$

$$A_1 = \int_0^{h^2} \sqrt{y} - \frac{y}{2} \cdot dy$$

$$A_2 = \int_0^h x^2 - f(x) \cdot dx$$

$$\int_0^{h^2} \sqrt{y} - \frac{y}{2} dy = \int_0^h x^2 - f(x) \cdot dx$$

$$\left(h - \frac{h^2}{2}\right)(2h) = h^2 - f(h)$$

$$f(h) = h^3 - h^2 \Rightarrow \boxed{f(x) = x^3 - x^2}$$

DETERMINATION OF FUNCTION

An equation in the form of $f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots\right) = 0$ is known as differential equation. i.e it contains one independent variable, one dependent variable and derivative of independent variable w.r.t. dependent variable.

eg:- $\frac{dy}{dx} = x + y$, $\frac{d^2y}{dx^2} + y = 0$, $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$

ORDER OF DIFFRENTIAL EQUATION

The highest order derivative present in the D.E. is k/n as order of D.E.

$\frac{dy}{dx} + x = 0$ — 1 $\qquad$ $\left(\frac{dy}{dx}\right)^2 + x = 0$ — 1

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 + xy = 0$ — 2 $\qquad$ $\sqrt{\frac{d^2y}{dx^2}} + y = 0$ — 2

Order of D.E is always a Natural Number.

DEGREE OF DIFFERENTIAL EQUATION

The highest power of highest order derivative provided in the polynomial form i.e, all the derivatives are in the polynomial form i.e. all the power of all the derivatives are finite whole Number.

	Order	Degree
$\frac{dy}{dx} + x^2 = 0$	1	1
$\sqrt{\frac{dy}{dx}} = x$	1	1
$\sqrt{dy/dx} + 2 = 0$	1	not defined

Question

Find degree and Order of DE :–

1. $\sqrt{\frac{d^2y}{dx^2} + 3} = 3$

$$\frac{d^2y}{dx^2} = 9 - 3$$

Order = 2
degree = 1

2. $\sqrt{\frac{d^2y}{dx^2} + 3} = \left(\frac{dy}{dx}\right)^{\sqrt{3}}$

$$\frac{d^2y}{dx^2} + 3 = \left(\frac{dy}{dx}\right)^{2\sqrt{3}}$$

Order = 2
degree = N.A.

3. $\sqrt{\frac{d^2y}{dx^2} + 3} = \left(\frac{dy}{dx}\right)^{1/3}$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{2/3 - 3}$$

Order = 2
degree = 3

4. $\frac{d^2y}{dx^2} = \sin\left(\frac{dy}{dx}\right)$

Order = 2
degree = N.D.

5. $\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) - \sin^2 y = 0$

Order = 1
degree = 2

6. $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$

Order = 1
degree = N.D.

7. $\frac{d^2y}{dx^2} = \cos x + \sin x$

Order = 2
degree = 1

8. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

Order = 2
degree = N.D.

9. $\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$

Order = 2
degree = 1

10. $(y')^{1/2} + (y'')^{2/3} + (y''')^{1/3} = 0$

Order = 3
Degree = N.D.

11. $x = e^{dy/dx}$

Order = 1
degree = 1

12. $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

Order = 1
degree = 1

13. $\ln\left(\frac{d^2y}{dx^2}\right) = \frac{1}{2}(x^2 + y)$

degree = 1
Order = 2

14. $\frac{dy}{dx} + x = \frac{1}{dy/dx}$

$$\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = 1$$

degree = 2
Order = 1

15. $\sqrt{d^2y/dx^2} = 4\sqrt{dy/dx}$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

Order = 2
degree = 2

QUESTION $\frac{d^2y}{dx^2} = \left[4 + \left(\frac{dy}{dx}\right)^6\right]^{1/4}$

Order = 2
degree = 4

QUESTION $e^{d^3x/dx^3} = \frac{x \cdot d^2y}{dx^2} - y$

order = 3
degree = N.D

$$\frac{d^3x}{dx^3} = \ln\left(x\, d^2y\, (dx^2 - y)\right)$$

QUESTION $\ln\left(\frac{dy}{dx}\right) = ax + by$

order = 1
degree = 1

QUESTION $\left(\frac{d^3x}{dy^3}\right) + 4 - \frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right) = 0$

Order = 3
degree = 2

FORMATION OF DIFFERENTIAL EQUATION

RELATION BETWEEN FAMILY OF CURVE AND DIFFERENTIAL EQUATION

A family of curve is an eqn of the type $f(x, y, c_1, c_2, \ldots\ldots c_n)$ where, $c_1, c_2 \ldots\ldots c_n$ are parameters and the above eqn is called family of curve of 'n' parameter.

eg:- $y = c_1 x$, $\quad y = c_1 x + c_2$, $\quad x^2 + y^2 = c_1$

Corresponding to any family of curve of 'n' parameter there exist a D.E of order 'n' which can be obtained by differentiating the curve 'n' time and eliminating all the parameter.

7n4

Question $y = mx \Rightarrow \frac{dy}{dx} = m \Rightarrow \frac{dy}{dx} = y/x$

Question $y = Ae^{x} + Be^{-x} \Rightarrow \frac{dy}{dx} = Ae^{x} - Be^{-x}$

$$\frac{d^2y}{dx^2} = Ae^{x} + Be^{-x} = \frac{d^2y}{dx^2} = y$$

Question

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \qquad \frac{B^2x^2}{A^2} + y^2 = B^2$$

$$\frac{2x}{A^2} + \frac{2y}{B^2}\frac{dy}{dx} = 0$$

$$\frac{2}{A^2} + \frac{2}{B^2}\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} = 0$$

$$\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = \frac{-\cancel{2}/A^2}{\cancel{2}/B^2} = -\frac{B^2}{A^2}$$

$$= +\frac{4}{x}\frac{dy}{dx}$$

$$\boxed{\left(\frac{dy}{dx}\right)^2 + y\left(\frac{d^2y}{dx^2}\right) = \frac{y}{x}\left(\frac{dy}{dx}\right)}$$

Question Find the DE of family of circle passing through origin and having centre on y-axis

$$(x-0)^2 + (y-k)^2 = k^2$$

$$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} = \frac{k\,dy}{dx}$$

$$1+\left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}=k.\frac{d^2y}{dx^2}$$

$$1+\left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}=\left(x+y\frac{dy}{dx}\right)\left(\frac{dx}{dy}\right)\left(\frac{d^2y}{dx^2}\right)$$

$$\boxed{1+\left(\frac{dy}{dx}\right)^2=x\left(\frac{dx}{dy}\right)\left(\frac{d^2y}{dx^2}\right)}$$

POINT TO REMEMBER...

- A line is a two parameter family of curve.
- A circle is a three parameter family of curve
- A parabola is a 4 parameter family of curve.
- Eclipse and hyperbola are 5 parameter family of curve.
- The Rectangular hyperbola is a 4 Parameter family of curve.

Question Order of DE of family of line $\parallel^{r}$ to y-axis.

(Slope block)

→ 1 Parameter Order = 1

Question Order of DE of family of circle having the circle $x^2+y^2=1$.

→ 2 Parameter (radius block according to centre)

Order = 2

Question Order of DE of family of parabola having y- axis as directrix.

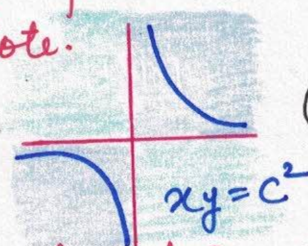
$\longrightarrow$ 2 (directrix fix . 2 Parabola)

Question Order of DE of family of ellipse having focii on x-axis.

$\longrightarrow$ 3 (y coordinate of focus fix, slope of directrix fix)

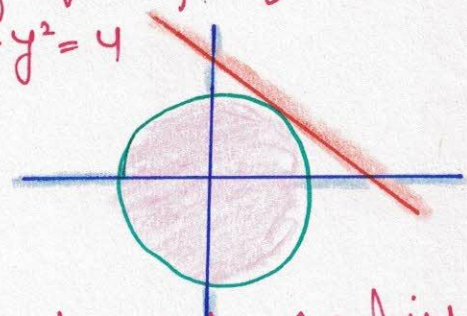
Question family of hyperbola having coordinate xy as asymptote.

$\longrightarrow$ 1



(Rectangular hyperbola)

Question Find the order of DE of family of curve line touching the circle $x^2+y^2=4$

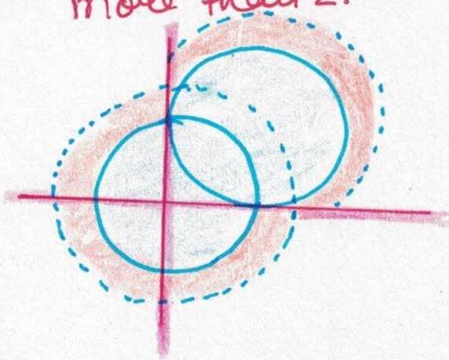


$y = mx \pm 2\sqrt{1+m^2}$

Parameter = 1

Order = 1

Question Order of DE of family of circle radius more than 2.



SOLUTION OF DIFFERENTIAL EQUATION

Solution of DE means retrieving the family of curve whose D.E is given. i.e a solⁿ of n^{th} order DE will be an n parametric family of curve.

TYPE-I VARIABLE SEPARATION METHOD

If a DE of order 1, degree 1 can be written as $\frac{dy}{dx} = \frac{f(x)}{g(y)}$, where $f(x)$ is a function of x alone and $g(y)$ is a function of y alone, then we can use method of variable separation to retrieve the family of curve as follow.

$$\int f(x)\cdot dx = \int g(y)\cdot dy$$

Question

Solve $x + y\frac{dy}{dx} = 0$

$$\int x\cdot dx = \int -y\cdot dy$$

$$\frac{x^2}{2} = \frac{-y^2}{2} + c \Rightarrow$$

$$\boxed{x^2 + y^2 = k}$$
family of circle

Question

Solve $y + x\frac{dy}{dx} = 0$

$$x\frac{dy}{dx} = -y \Rightarrow -\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$= -\ln y = \ln x + c$$

$$c = \ln x + \ln y = \ln(xy) = c$$

$$xy = e^c \qquad \boxed{\Rightarrow xy = k}$$

Question

$$\frac{dy}{dx} = e^{x+y}$$

$$dy = e^{x+y} \qquad dy = e^{x}.e^{y}.dx$$

$$\int e^{-y}dy = \int e^{x}dx$$

$$-e^{-y} = e^{x} + c \Rightarrow \boxed{x + \frac{1}{e^{y}} = c}$$

Question

$$\frac{dy}{dx} + \sqrt{1+x^2+y^2+x^2y^2} = 0$$

$$x^2(1+y^2) + 1(1+y^2)$$

$$\frac{dy}{dx} = -\sqrt{(x^2+1)\ (y^2+1)}$$

$$\int \frac{dy}{\sqrt{1+y^2}} = \int \sqrt{1+x^2}\ .dx$$

$$= \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|$$

$$\boxed{\ln\left|1+\sqrt{1+y^2}\right| = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right| + c}$$

TYPE-II REDUCIBLE TO VARIABLE SEPARATION

If the D.E is of the type $dy/dx = f(ax+by+c)$ then, it is a D.E which can be reduced to Variable separation by substituting $ax+by+c=t$

Question

$$\frac{dy}{dx} = (x+2y+1)^2$$

$$x+2y+1 = t \qquad 1 + 2\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = (t)^2 \qquad \frac{1}{2}\left(\frac{dt}{dx} - 1\right) = t^2$$

$$\frac{dt}{dx} - 1 = 2t^2$$

$$\frac{dt}{dx} = 2t^2+1 \Rightarrow \int \frac{dt}{2t^2+1} = \int dx$$

$$\frac{1}{2}\left(\tan^{-1}\sqrt{2}t\right) = x + c$$

$$\boxed{\frac{1}{2}\tan^{-1}\sqrt{2}\,(x+2y+1) = x+C}$$

Question $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

$x+y=t \qquad 1+\frac{dy}{dx} = \frac{dt}{dx} \qquad \frac{dy}{dx} = \frac{dt}{dx} - 1$

$$\frac{dt}{dx} = \cos t + \sin t + 1$$

$$\sqrt{2}\int \frac{dt}{\frac{\cos t + \sin t + 1}{\sqrt{2}}} = \int dx$$

$$\frac{1}{\sqrt{2}}\int \frac{dt}{\sin(t+\pi/4) + \sin\pi/4} = x + C$$

$$\boxed{\frac{1}{\sqrt{2}}\int \frac{dt}{2\sin\left(\frac{t+\pi/2}{2}\right)\cos(t/2)} = x+C}$$

Question Find the solution of a D.E passing through (0,0) in the form of $y=f(x)$ of $\frac{dy}{dx} = \sin(10x+6y)$

$$10x+6y=t$$

$$10 + 6\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} = 6(\sin t) + 10$$

$$\frac{1}{2}\int \frac{dt}{3\sin t + 5} = \int dx$$

$$\frac{1}{2}\int \frac{dt\ \ (x^2+t/2)}{3(2\tan t/2)+5\tan^2 t/2+5}$$

$$\frac{1}{2}\int \frac{2\,dz}{6z+5z^2+5} \qquad 5z^2+6z+5$$

$$\frac{1}{2}\int \frac{6\,dz}{5\left(z^2 \frac{1}{2}\times 5\times 2\left(6z+\frac{36}{100}\right)-\frac{36}{20}+5\right.}$$

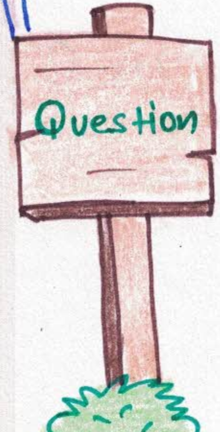
$$\int \frac{dz}{5(z+6/5)^2+\left(\sqrt{8/20}\right)^2} = x+c$$

$$\frac{1}{5}\,\frac{\sqrt{20}}{8}\tan^{-1}\frac{(z+6/5)\,20}{8} = x+c$$

$$\frac{1}{5}\,\frac{\sqrt{20}}{8}\tan^{-1}\left(\frac{\tan(5x+2y)}{8}+\frac{6}{5}\right)20 = x+c$$

$$\frac{\sqrt{20}}{40} \qquad \frac{3\sqrt{20}}{40}$$

$$\boxed{\frac{1}{4}\tan^{-1}\left(\frac{5\tan(5x+340)}{4}\right) = x+\frac{1}{4}\tan^{-1}\frac{3}{4}}$$



Bridge b/w Homogeneous and variable separation.

$$\frac{dy}{dx} = \frac{4x+6y-1}{6x+9y+2}$$

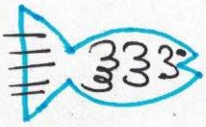
$$\frac{\cancel{6x}+6y+2\ -2x}{6\cancel{x}+9y+2\ -3y-3} \quad 6x+9y+2$$

$$\frac{4x+6y-1}{6x+9y+2} = \frac{2(2x+3y)-1}{3(2x+3y)+2}$$

$$2x+3y\ t \Rightarrow 2+\frac{3y}{dx} = \frac{dt}{dx}$$

$$\frac{2t-1}{3t+2} = \frac{1}{3}\left(\frac{dt}{dx}-2\right)$$

$$\frac{6t-3+6t+4}{3t+2} = \frac{dt}{dx} = \frac{12t+1}{3t+2}$$



$$\int \frac{(12+1)\,dt}{3t+2} = \int dx$$

$$\boxed{\int \frac{3t+2}{12t+1}\,dt = \int dx}$$

TYPE III — HOMOGENEOUS DIFFERENTIAL EQUATION

A D.E which is of the type $\frac{dy}{dx} = f(y/x)$ is k/as Homogeneous D.E and to obtain the solution of such D.E substitute $(y/x) = t$ i.e, $y = xt$.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{(y/x)^2 - 1}{2(y/x)}$$

$$y/x = t \qquad \frac{dy}{dx} = \frac{x\,dt}{dx} + t$$

$$\frac{x\,dt}{dx} = -\frac{t^2-1}{2t}$$

$$-\int \frac{2t\,dt}{t^2+1} = \int \frac{dx}{x} \qquad -\ln(t^2+1) = \ln x + c$$

$$\boxed{c = \ln x\left(\left(y/x\right)^2 + 1\right)}$$

Question

$$x\left(\cos(y/x)\cdot\frac{dy}{dx}\right) = y\cdot\cos(y/x) + k$$

$$\frac{y}{x} = t$$

$$\left(x\frac{dt}{dx} + t\right) = \frac{t\cos t}{\cos t} + \frac{1}{\cos t}$$

$$x\frac{dt}{dx} = t + \sec t$$

$$\int\frac{dt}{\sec t} = \int\frac{dx}{x}$$

$$\int\cos t = \ln x$$

$$\sin t = \ln x + c$$

$$\boxed{\sin y/x = \ln + c}$$

Question

$$(3xy + y^2) + (x^2 + xy)\,dy/dx = 0$$

$$3(y/x) + (y/x)^2 + \left(1 + (y/x)\right)\left(x\,dt + 1\right)$$

$$-(t^2 + 3t) = (1+t)\left(1 + x\frac{dt}{dx}\right)$$

$$-\left[\frac{t(t+3)+t}{1+t}\right] = x\frac{dt}{dx}$$

$$\int\frac{dx}{x} = \int\frac{(1+t)\cdot dt}{t^2+4t}$$

$$\boxed{\ln x = \int\frac{1+t}{4t+2t^2}}$$

$\Rightarrow (2t + 4) + \lambda$

$2 + \lambda = 1 \qquad 4 + \lambda = 1$

$\lambda = -1 \qquad \lambda = -3$

TYPE - IV

REDUCIBLE TO HOMOGENEOUS

Question

$$\frac{dy}{dx} = \frac{x+y}{x-y+2}$$

It is reducible to Homogeneous equation and to obtain the solution of such equation Subtitute $x = X+h$, $y = Y+k$.

$$\frac{dy}{dx} = \frac{X+h+Y+k}{X+h-Y-K+2}$$

$$= \frac{X+Y+h+k}{X-Y+h-k+2}$$

$$h+k=0$$
$$h-k+2=0$$
$$2h+2=0$$
$$h=-1$$
$$k=1$$

$x = X-1$ $\qquad$ $y = Y+1$

$$\frac{dy}{dx} = \frac{X+Y}{x-Y} = \frac{1+Y/X}{1-Y/X}$$

$\frac{Y}{X} = t$ $\qquad$ $\boxed{\frac{dy}{dx} = \frac{1+t}{1-t}}$

Question

$$\frac{dy}{dx} = \frac{x+y-3}{2x-3y+4}$$

$x = X+h$
$y = Y+k$

$$\frac{dy}{dx} = \frac{X+h+Y+k-3}{2X+2h-3Y-3K+4}$$

$$3(h+k-3=0)$$
$$2h-3K+4=0$$
$$5h-5=0$$
$$h=1 \qquad K=2$$

$$\frac{dy}{dx} = \frac{X+Y}{2x-3Y}$$

$$\boxed{\frac{dy}{dX} = \frac{1+(Y/X)}{2-3(Y/X)}}$$

Question

$$(x^2+4y^2+4xy)\,dy = (2x+4y+1)\cdot dx$$

$$(x+2y)^2\cdot dy = (2(x+2y)+1)\cdot dx$$

$$x+2y = t \qquad 1+2\frac{dy}{dx} = \frac{dt}{dx}$$

$$t^2\left(\frac{dy}{dx}\right) = (2t+1)$$

$$\frac{t^2}{2}\left(\frac{dt}{dx}-1\right) = 2t+1$$

$$\frac{dt}{dx} = \frac{4t+2+t^2}{t^2}$$

$$\boxed{\int \frac{t^2\,dt}{4t+2+t^2} = \int dx}$$

$t^2 = t^2+4t$

TYPE-V

LINEAR DIFFERENTIAL EQUATION...

A D.E in the form of $\frac{dy}{dx} + P(x)\cdot y = Q(x)$ where $P(x)$ and $Q(x)$ are function of 'x' alone. is known as linear D.E or standard D.E.

NOTE

$\frac{dx}{dy} + P(y)\cdot x = Q(y)$ is also a standard D.E

The solution of linear D.E will be done in 2 parts

Find the integrating factor

Integration factor $e^{\int P(x)\cdot dx}$

but we don't put integration constant.

Solution of D.E will be

$$y(IF) = \int Q(x)\,(IF)\cdot dx$$

Question

$$x^2\frac{dy}{dx} - xy = 2$$

$$\frac{dy}{dx} - \frac{y}{x} = \frac{2}{x^2}$$

$$P(x) = -\frac{1}{x} \qquad q(x) = \frac{2}{x^2}$$

$$IF = e^{\int 1/x \cdot dx}$$

$$IF = e^{-\ln x} = +1/x$$

$$y\left(\frac{1}{x}\right) = \int \frac{2}{x^2}\cdot\frac{1}{x}\,dx$$

$$\frac{y}{x} = \frac{\cancel{2}(x^{-2})}{-\cancel{2}} + c \Rightarrow \boxed{\frac{y}{x} = \frac{1}{x^2} + c}$$

Question

$$\frac{dy}{dx} + y\cos x = \sin 2x$$

$$IF = e^{\int \cos x \cdot dx}$$

$$IF = e^{\sin x}$$

$$\sin x = t$$

$$\cos x \cdot dx = dt$$

$$(y)(e^{\sin x}) = \int \sin 2x\, e^{\sin x}\cdot dx$$

$$y e^{\sin x} = \int 2t \cdot e^t \cdot dt$$

$$y e^{\sin x} = 2\left[e^t t - \int 1 e^t \cdot dt\right]$$

$$\boxed{y e^{\sin x} = 2\left(e^{\sin x}\sin x - e^{\sin x}\right)}$$

Question

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

$$IF = e^{2\int x\,dx/1+x^2} = e^{\ln(1+x)} = (1+x^2)$$

$$y(1+x^2) = \int\left(\frac{4x^2}{1+x^2}\right)(1+x^2)\cdot dx$$

$$\boxed{y(1+x^2) = \frac{4x^3}{3} + c}$$

Question

$$\frac{dy}{dx} + (\sec^2 x)(y) = \tan x \sec^2 x$$

$$e^{\int \sec^2 x\, dx} = e^{\tan x + \sec x}$$

$$(e^{\tan x})y = \int (e^{\tan x})(\tan x \sec^2 x)\, dx$$

$$\int e^t (t)(1+\cancel{t^2})\frac{dt}{1+\cancel{t}}$$

$$(e^{\tan x})y = (e^t - e^t)$$

$$\boxed{(e^{\tan})y = \tan x\, e^{\tan x} - e^{\tan x} + c}$$

Question

$$\frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}$$

$$e^{\int \frac{1}{1+y^2} \cdot dy} = e^{\tan^{-1} y}$$

$$(e^{\tan^{-1} y})x = \int (e^{\tan^{-1} y})\left(\frac{\tan^{-1} y}{1+y^2}\right) dy$$

$$(e^{\tan^{-1} y})x = \int e^t\, t\, dt$$

$$\boxed{(e^{\tan^{-1} y})x = (\tan^{-1} y)e^{\tan^{-1} y} - e^{\tan^{-1} y} + c}$$

Question

$$x(x^2+1) \cdot \frac{dy}{dx} = y(1-x^2) + x^2 \ln x$$

$$\frac{dy}{dx} - y\left(\frac{(1-x^2)}{x(x^2+1)}\right) = \frac{x^2 \ln x}{x(x^2+1)}$$

$x + x^2 = t$

$1 + 3x^2 dx = dt$

$$IF = e^{-\int \frac{(1-x^2)\cdot dx}{x(1+x^2)}} = e^{\int \frac{1+3x^2\cdot dx}{x(1+x^2)} - \frac{4x^2}{x(1+x^2)}\cdot dx}$$

$$= e^{\ln(x(1+x^2)) - \ln(1+x^2)^2}$$

$$\boxed{= e^{-\ln \frac{x(1+\cancel{x^2})}{1+x^2}} = \frac{x}{1+\cancel{x^2}}}$$

Question

$$\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$$

$$\frac{1}{\cos y(x + 2\sin y)}$$

$$\left(\frac{dy}{dx}\right)\cos y = \frac{dx}{x + 2\sin y}$$

$$\frac{dx}{dy} = x\cos y + \sin 2y$$

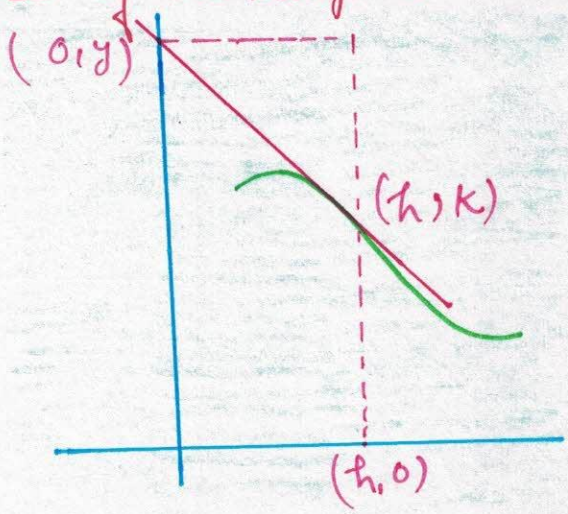
$$IF = e^{-\int \cos y \cdot dy} = e^{-\sin y}$$

$$\left(e^{-\sin y}\right) x = \int \left(e^{-\sin y}\right)(2\sin y\cos y)\cdot dy$$

$$\boxed{\left(e^{-\sin y}\right)\cdot x = 2\int (e^t)\, t\cdot dt}$$

Question

Find the curve such that the area of the rectangle constructed on the axis of any pt and the initial coordinate of the tangent at this Point is 4.



$$y - k = \frac{dy}{dx}(x-h)$$

$$y = k - \frac{dy}{dx}(h)$$

$$area = xy = 4$$

$$x\left(k - \frac{dy}{dx}h\right) = 4$$

$$\cancel{kx - xh\frac{dy}{dx}} = 4 \qquad\qquad \cancel{\frac{kx-y}{x}} = \frac{h\,dy}{dx}$$

$$xy - x^2\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} - \frac{xy}{x^2} = \frac{-4}{x^2}$$

$$IF = e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

$$\left(\frac{1}{x}\right)y = \int\left(\frac{1}{x}\right)\left(\frac{-4}{x^2}\right)\cdot dx$$

$$\frac{y}{x} = \frac{2\cancel{4}\,x^{-2}}{\cancel{2}} + c$$

$$\frac{y}{x} = \frac{2}{x^2} + cx^2$$

$$\boxed{xy = cx^2 + 2}$$

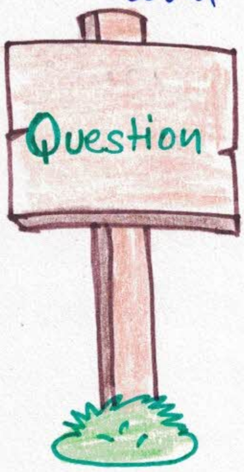
Given a f^n g, which has a derivative $g'(x)$ for every real 'x' and which satisfy $g'(0)=2$ and $g(x+y) = e^y.g(x) + e^x.g(y)$ for all x and y. find $g(x)$ and determine the range of f^n. Also Compute the area bounded by $g(x)$ and x-axis.

EQUATION REDUCIBLE TO LINEAR DIFFERENTIAL EQ^n

BERNOULI'S EQUATION

$$\frac{dy}{dx} + P(x)\cdot y = Q(x)\cdot y^n$$

where $P(x)$ and $Q(x)$ are f^n of (x) and it is reducible to linear form by dividing it by y^n and then substitute $y^{(-n+1)} = z$



$$\frac{dy}{dx} + xy = x^3y^3$$

$$\frac{dy}{y^3\,dx} + \frac{x}{y^2} = x^3$$

$y^{-2} = t$

$-2y\,y^{-3}dy = dt$

$$\frac{-1\,dt}{2\,dx} + xt = x^3$$

$$\frac{dt}{dx} - 2xt = -2x^3$$

$$e^{-2\int x\,dx} = e^{-2x^2/2} = e^{-x^2}$$

$$(e^{-x^2})(t) = \int e^{-x^2}(-2x^3)\,dx$$

$-x^2 = z$

$-2x\,dx = dz$

$$(e^{-x^2})(t) = -\int e^{z}(z)\,dz$$

$$(e^{-x^2})(t) = -\left[e^{z}z - e^{z}\right]$$

$$(e^{-x^2})(y^{-2}) = -\left[e^{-x^2}(-x^2) - e^{-x^2}\right]$$

$$\boxed{(y^{-2}) = x^2 + 1}$$

Question $\frac{dy}{dx} + \frac{y}{x}\ln y = \frac{y(\ln y)^2}{x^2}$

$$\frac{dy}{dx} + \frac{y}{x} \times \frac{1}{(\ln y)} = \frac{1}{x^2}$$

$$\frac{1}{\ln y} = t$$

$$\frac{-(\ln y)^{-2}\,dy}{y} = dt$$

$$y(\ln y)^2 + \frac{dt}{dx} + \frac{t}{x} = \frac{-1}{x^2}$$

$$e^{\int 1/x \cdot dx} = e^{\ln x} = x$$

$$\boxed{(x)t = \int (x)\left(-\frac{1}{x^2}\right) \cdot dx}$$ can solve further

Question $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1+y^2) = 0$

$$\frac{dy}{dx} + 2x\tan^{-1}y - x^3 + 2xy^2 - x^3y^2$$

$$\tan^{-1}y = t$$

$$\left(\frac{1}{1+y^2}\right)\frac{dy}{dx} = \frac{dt}{dx}$$

$$x^2 = t$$

$$x\,dx = \frac{dt}{2}$$

$$(1+y^2)^{dy/dx} + (2xt - x^3) = 0$$

$$\frac{dt}{dx} + (2x)t = x^3$$

$$e^{\int 2x\,dx} = e^{x^2} = IF$$

$$(e^{x^2})(t) = \int (e^{x^2})\cdot(x^3)\cdot dx$$

$$e^{x^2}(t) - \frac{1}{2}\int e^{t} \cdot t\,dt$$

$$2e^{x^2}(t) = e^{x^2}x^2 - e^{x^2} + c$$

$$\boxed{2e^{x^2}(1+y^2) = x^{e^{x^2}} - e^{x^2} + c}$$

EXACT DIFFERENTIAL EQUATION

following are the exact D.E which must be remember.

- $x\,\frac{dy}{1} + y\,dx = d(xy)$
- $\frac{y\,dx - x\,dy}{y^2} = d(x/y)$
- $\frac{x\,dy - y\,dx}{x^2} = d(y/x)$
- $\frac{x\,dy + y\,dx}{xy} = d(\ln xy)$
- $\frac{dx + dy}{x+y} = d(\ln(x+y))$
- $\frac{x\,dy - y\,dx}{xy} = d(\ln y/x)$
- $\frac{y\,dx - x\,dy}{xy} = d\left(\ln \frac{x}{y}\right)$
- $\frac{x\,dy - y\,dx}{x^2+y^2} = d\left[\tan^{-1}\left(\frac{y}{x}\right)\right]$

Question

$y\cos(y/x)(x\,dy - y\,dx) + x\sin(y/x)(x\,dy + y\,dx) = 0$

Divide by x^2

$y\cos(y/x)\,d(y/x) + x\sin(y/x)\,d(xy) = 0$

$y\cot(y/x)\,d(y/x) + \frac{d(xy)}{x} = 0$

$\cot(y/x)\,d(y/x) + \frac{d(xy)}{xy} = 0$

$$\boxed{\ln\left|\sin(y/x)\right| = -\ln|xy + c}$$

Question

$$x^4\frac{dy}{dx} + x^3y + \operatorname{cosec} xy = 0$$

$$x\,dy + y\,dx + (\operatorname{cosec} xy)\frac{dx}{x^3} = 0$$

$$d(xy) = -\operatorname{cosec}(xy)\frac{dx}{x^3}$$

$$\int \sin(y)\,d(xy) = -\int\frac{dx}{x^3}$$

$$\boxed{-\cos xy = \frac{x^{-2}}{2} + c}$$

Question

$$\left[(x)(y^4) + y\right]\cdot dx - x\,dy = 0$$

$$\frac{xy^4}{y^2}dx + \frac{y\,dx - x\,dy}{y^2} = 0$$

$$y^2x\,dx + d(x/y) = 0$$

$$d(x/y) = -xy^2\,dx$$

$$\boxed{x^3dx + \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right) = 0}$$